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Local Probability of Severe Hail Equations for the WFO Columbia, SC County Warning Area

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ABSTRACT

In the late 1990's, a probability of severe hail equation (LPSH75) for diameter greater than or equal to 0.75 in. (1.9 cm) was developed for the Columbia South Carolina County Warning Area (CAE CWA) using a logistic regression methodology. This equation provided a "VIL of the Day" which forecasters used operationally as an aid in the warning decision making process. In early 2010, National Weather Service (NWS) criterion for severe hail was changed to a diameter greater than or equal to 1.00 in. (2.5 cm). The purpose of this study was to develop new objective methods to estimate the probability of severe hail based on the revised NWS definition. In addition to a new probability of severe hail equation (LPSH100) for diameter greater than or equal to 1.00 in. (2.5 cm), two equations (LPSH0 and LPSH-20) were developed that relate the height of the 50dBZ core of a particular thunderstorm to the $0^{o}C$ and $-20^{o}C$ levels to estimate the probability of severe hail. LPSH0 and LPSH-20 can be useful before convection develops and in conjunction with radar storm interrogation by providing an objective method to estimate the probability of severe hail in near real time. Local applications were developed to display the severe hail probability output to the forecasters in the Advanced Weather Interactive Processing System (AWIPS).

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1. Introduction

DeLisi (1998) derived a probability of severe hail equation (LPSH75) with a diameter greater than or equal to 0.75 in (1.9 cm) for the Columbia South Carolina County Warning Area (CAE CWA). This equation has been successful as an aid to the forecasters in the warning decision making process by providing a "VIL of the Day" and enhancing confidence in severe hail detection. The National Weather Service (NWS) criterion for severe hail was changed in 2010 to a diameter greater than or equal to 1.00 in (2.5 cm), thus the LPSH75 was no longer valid for warning operations. The purpose of this study was to develop new objective methods to estimate the probability of severe hail based on the revised NWS definition, and aid the forecasters in the warning decision making process.

A similar approach to DeLisi (1998) was used to derive local probability of severe hail equations for the CAE CWA by using the logistic regression technique. In addition to a new equation intended to provide an estimate of a "VIL of the Day" prior to convective development (LPSH100), two additional equations (LPSH0 and LPSH-20) were derived that estimate the probability of severe hail by relating the height of the 50dBZ core of a particular thunderstorm to the 0^{o}C and -20^{o}C isotherm levels. Frugis and Wasula (2011), Donavon and Jungbluth (2007) and Kramar and Waters (2009) suggested that confidence of severe hail within a thunderstorm can be increased by examining the height of the 50 dBZ core as it relates to the 0^{o}C and -20^{o}C isotherm levels. In these studies, the height of storm reflectivity cores (greater than or equal to 50 dBZ) were strongly correlated with severe hail (diameter 1.00 in (2.5 cm) or greater) and statistical methods developed proved useful in operations. This is likely because the 50 dBZ core has been shown to be a good indicator of updraft strength (Donovan and Jungbluth 2007). LPSH0 and LPSH-20 were intended to be used prior to convection developing and in conjunction with radar storm interrogation.

Predictor variables included in the LPSH100 are vertically integrated liquid (VIL), the ratio of VIL to the echo top height commonly known as the VIL density (VILD), both computed from the WSR-88D (Amburn and Wolf 1997); the Total Totals index (TT), and the 500-hPa Temperature (H5), both computed from the RUC model forecast data. It should be noted that the "VIL of the Day" is taken to be the value of the VIL in the LPSH100 equation that together with the other predictors specified in the equation results in a probability of severe hail equal to 40 percent. This probability threshold was determined through an analysis of probability of detection (POD), false alarm rate (FAR), and the critical success index (CSI). This analysis is discussed in detail in section 3. The predictor variables for LPSH0 and LPSH-20 include the VILD, H5, and the height of the 50 dBZ core of the particular storm of interest. The 0^{o}C isotherm level was included as a predictor variable for LPSH0 and the -20°C isotherm level was included as a predictor variable for LPSH-20. The dependent variable is the probability of severe hail.

2. Data and Methodology

The incidence of severe hail was represented by the dependent variable (categorical binary predictand) for all the equations. These data were derived from Local Storm Reports (LSR's) for the CAE CWA from February, 2007 through April, 2011. Initially, there were 239 cases in the developmental dataset used to derive the equations. These cases included 67 incidences of no hail, 80 were of small hail (diameter less than 1.00 in (2.5 cm)), and 92 were of severe hail (diameter 1.00 in (2.5 cm) or greater). Incidences of no hail were determined by arbitrarily selecting storms for which there were no storm reports. During the equation development the dataset was reduced to 221 cases for LPSH100 and 224 cases for both LPSH0 and LPSH-20 after outliers were removed.

As discussed in Frugis and Wasula (2011) and DeLisi (1998), there are intrinsic issues associated with developing a hail database that can result in potential sources of error. The practice of relying on public spotters, many who are not trained to give accurate measurements, locations, and time of the event, may result in inaccurate hail reports. In addition, the majority of the CAE CWA is rural which further complicates the issue with hail producing thunderstorms possibly going unreported. In order to minimize the sources of these errors, reports from the database were discarded if the storm structure analysis determined that the report was not in the location where the maximum expected hail size would likely occur and the hail reports were not within four volume scans subsequent to the time of the maximum 50 dBZ core (Kramar and Waters 2009).

Outliers were removed from the developmental data set and were likely the result from a misclassification of the binary predictor variable (sub-severe versus severe hail cases). Outliers are cases with large residuals and can reduce the overall fit of the model. Examination of the standardized deviance residuals was performed to identify potential outliers. Standardized deviance residuals larger than 2.0 or smaller than -2.0 were investigated as to whether they should be considered as outliers and removed. This process lead to the removal of 18 outliers from the developmental dataset used for LPSH100 resulting in 221 cases. Likewise, 15 outliers were removed from the developmental dataset used for LPSH0 and LPSH-20 resulting in 224 cases.

Potential predictor variables included radar products from the Columbia, SC (KCAE) WSR-88D and parameters from the Rapid Update Cycle (RUC) model and Storm Prediction Center (SPC) meso-analysis. A complete list can be found in Appendix A. The list was similar to DeLisi (1998) except for the additional variables from the SPC meso-analysis. In addition, DeLisi (1998) used observed sounding data from Peachtree City Georgia (FFC) to acquire the potential environmental predictors. In this study, RUC model forecast sounding data using a local BUFKIT archive was chosen for most of the environmental predictors. Since the SPC meso-analysis is derived from the 40-km RUC model, several of the parameters from that archive were chosen as potential predictors.

After reviewing storm reports through the SPC severe thunderstorm events archive (CAE LSR's), the identified thunderstorms were interrogated using the National Climatic Data Center (NCDC) Weather and Climate Toolkit and the commercially available software package GR2 Analyst (Gibson 2011). KCAE WSR-88D potential predictor variables came from the storm interrogation during the time of maximum VIL. In all the cases, to ensure the pre-storm environment was captured and to limit convective contamination, a RUC model forecast time-step at least 1-hr prior to the hail report was used. In addition, BUFKIT stations closest to the hail report were used and only one hail report (the maximum size reported) per thunderstorm was included in the dataset. This data collection process was also used for the thunderstorm cases that did not produce hail. No cases were selected that were within 23-km of the KCAE WSR-88D to minimize the negative effects on the data quality from the radar's "cone of silence." The maximum distance used from the KCAE WSR-88D was 153-km.

Pearson's product-moment correlation coefficients (r) between all the potential predictor variables and the dependent variable were calculated to measure the linear dependence. In addition, box and whisker plots were created to further evaluate the potential predictors and the dependent variable. The potential predictors that had the strongest correlation coefficients were: The height of the 50 dBZ core above the 0^{o}C isotherm level (*hgt50dbzfz*), the height of the 50 dBZ core above the -20 ^{o}C isotherm level (*hgt50dbz20*), VIL, and the VILD. The probability-value (p-value) for each of the correlations was zero, indicating that the probability of such a correlation by random chance was zero. The correlation statistics for each of these potential predictors are shown in Table 1.

The box and whisker plots comparing the distributions between these potential predictors and the occurrence and non-occurrence of severe hail are shown in Figures 1-4. For *hgt50dbzfz*, the cases of severe hail showed middle clustering of the data about the median and exhibited little in the way of skewness which indicated that the distribution was resistant to any outliers (Wilks 1995). The middle 50% of the ranked data was beyond the 75^{th} percentile of the non-occurrence cases with no overlap between the two distributions. The non-occurrence of severe hail cases also exhibited downward skewness which indicated outliers were less probable beyond the 50^{th} Percentile. This meant that the data limits were approached closer to the median in the positive direction and along with no overlap between the two distributions suggested that *hgt50dbzfz* was a potentially good predictor of severe hail. The *hgt50dbz20* box and whisker plots were very similar to the *hgt50dbzfz* plots. The severe hail cases exhibited slight downward skewness and 50% of the ranked data was beyond the 75^{th} percentile of the non-occurrence cases with no overlap between the two distributions. The non-occurrence cases exhibited pronounced downward skewness indicating a low probability of outliers beyond the 50^{th} percentile. Similar to the *hgt50dbzfz* and *hgt50dbz20* variables, the box and whisker plots for the VIL variable exhibited pronounced downward skewness for the non-occurrence cases indicating a low probability of outliers beyond the 50^{th} percentile and no overlap between the distributions. The box and whisker plots for the VILD variable exhibited some upward skewness for the non-occurrence cases indicating a high probability of outliers beyond the 50^{th} percentile (data limits were approached closer to the median in the negative direction) but still no overlap between the middle 50% of the ranked non-occurrence cases and the middle 50% of the ranked severe hail cases (Banacos 2011).

With no overlap of the two distributions for each of the potential variables with the strongest correlation coefficients, it could be stated qualitatively that the *hgt50dbzfz*, *hgt50dbz20*, VIL, and VILD were all potentially good predictors for the occurrence of severe hail. Scatter plots were developed that showed the relationship between the height of the 50 dBZ core and the height of the -20^{o}C and 0^{o}C isotherms with regards to the severe hail cases in the developmental sample. The scatter plots indicated that a 50 dBZ core with a median height of 39,151 ft. and a mean of 38,049 ft. was associated with severe hail, which is consistent with the results from the Donavon and Jungbluth (2007), Frugis and Wasula (2011) and Kramar and Waters (2009) studies (Figs. 5 and 6). All these variables were eventually selected as predictors during the logistic regression model developmental process.

A statistical software package S-Plus (Martin 1988) that is commercially available was used to derive three probability of severe hail equations using the logistic regression technique. The logistic regression model is used to estimate the probability of a categorical occurrence by fitting measurable predictor variables to a logistic function (DeLisi 1998). A complete discussion of logistic regression can be found in Freeman (1987) and Wilks (1995). A probability of severe hail model can be expressed as:

$$\text{PSH}=100\times\left(1-\left(\frac{1}{(1+exp(B_0+B_1X_1+B_2X_2+_{\dots}+B_nX_n))}\right)\right) \quad \textbf{(1)}$$

where,

$\boldsymbol{PSH}$ = Probability of Severe Hail

$\boldsymbol{B_0}$ = Intercept

$\boldsymbol{B_1}_{\dots}\boldsymbol{B_n}$ = Regression Coefficients

$\boldsymbol{X_1}_{\dots}\boldsymbol{X_n}$ = Predictor Variables

The stepwise developmental methodology explained in DeLisi (1998) was used to derive the equations in this study. The likelihood ratio chi-square test (G^2) was used to assess whether adding the potential predictor variables to the model resulted in a decrease in the residual deviance that was statistically significant. The null hypothesis was that there was no statistically significant improvement in the model fit after adding a potential predictor variable. Each potential predictor variable was run in a one predictor variable model and tested for statistical significance. Models with high G^2 have little predictive skill, so the variable that resulted in the greatest reduction in G^2 had the most predictive value and was retained. In addition, a resulting p-value less than or equal to 0.05 was required to be considered statistically significant (at the 95 % confidence level) and rejection of the null hypothesis. The process continued by running all of the remaining variables in a two predictor variable model with the variable selected from the one predictor variable model. Again, the variable that resulted in the greatest reduction in G^2 was retained, provided that the associated p-value was less than or equal to 0.05. Additional variables were added to the model until none of the remaining variables produced a p-value less than or equal to 0.05.

All the potential predictor variables were considered for the LPSH100 equation except *hgt50dbzfz* and *hgt50dbz20*. All the potential predictor variables were considered for the LPSH0 equation except *hgt50dbz20* and all the potential predictor variables were considered for the LPSH-20 except *hgt50dbzfz*.

3. Results

The derived LPSH100 model was:

$$\text{PSH}=100\times(1-\frac{1}{1+\exp(-18.28+0.05X_1+0.20X_2+0.03X_3-0.43X_4)}) \quad \textbf{(2)}$$

where,

$\boldsymbol{PSH}$ = Probability of Severe Hail

$\boldsymbol{X_1}$ = VILD (0.003284 kg m^{-3})

$\boldsymbol{X_2}$ = VIL (kg m^{-2})

$\boldsymbol{X_3}$ = TT (^{o}C)

$\boldsymbol{X_4}$ = H5TEMP (^{o}C)

The p-values associated with the reductions in G^2 for each of the predictor variables selected in the LPSH100 were 0.0000, 0.0000, 0.0019, and 0.0000 respectively.

The derived LPSH0 model was:

$$PSH=100\times(1-\frac{1}{1+\exp(-14.76+0.0003X_1+1.12X_2-0.29X_3)})\ \textbf{(3)}$$

where,

$\boldsymbol{PSH}$ = Probability of Severe Hail

$\boldsymbol{X_1}$ = *hgt50dbzfz* (ft.)

$\boldsymbol{X_2}$ = VILD (0.003284 kg m^{-3})

$\boldsymbol{X_3}$ = H5TEMP (^{o}C)

The p-values associated with the reductions in G^2 for each of the predictor variables selected in the LPSH0 were 0.0000, 0.0139, and 0.0009 respectively.

The derived LPSH-20 model was:

$$PSH=100\times(1-\frac{1}{1+\exp(-12.43+0.0003X_1+1.17X_2-0.31X_3)})\textbf{(4)}$$

where,

$\boldsymbol{PSH}$ = Probability of Severe Hail

$\boldsymbol{X_1}$ = *hgt50dbz20* (ft.)

$\boldsymbol{X_2}$ = VILD (0.003284 kg m^{-3})

$\boldsymbol{X_3}$ = H5TEMP (^{o}C)

The p-values associated with the reductions in G^2 for each of the predictor variables selected in the LPSH-20 were 0.0000, 0.0078, and 0.0003 respectively.

One of the assumptions of logistic regression is that the predictor variables are not strongly correlated with each other (multicollinearity). Multicollinearity can result in biased coefficients and increased residual deviance (inflation) of the predictor estimates. The degree of multicollinearity between the predictor variables in each equation was examined using Pearson's product-moment correlation coefficient (r) (Table 2). For LPSH100, VIL and VILD were the only predictor variables strongly correlated. For LPSH0, the *hgt50dbzfz* and VILD predictor variables were strongly correlated. For LPSH-20, the *hgt50dbz20* and VILD predictor variables were strongly correlated. Each of the associations had a p-value of zero, which meant that the probability that the variables were strongly correlated by random chance was zero.

However, as stated in DeLisi (1998), multicollinearity in logistic regression is not as serious a problem as in multiple linear regression especially if the developmental sample size is large. As stated in Freeman (1987), the required developmental sample size required to satisfy the assumptions of logistic regression is ***n > 10(S + 1)***, where n corresponds to the sample size and S corresponds to the number of predictor variables in the model. Since the developmental sample size was 221 cases for LPSH100 and 224 cases for both LPSH0 and LPSH-20, it can be stated that the developmental sample for each of the models was sufficiently large. For all the equations in this study, the deviance decreased (deflation) as the predictor variables were added during the development. Therefore, it can be stated that the predictive power of the variables selected using the stepwise methodology outweighed any collinearity issues because the residual deviance decreased.

To facilitate the use of these equations in operations, three computer applications were developed and made available in AWIPS. Each application has a graphical user interface (GUI) that allows the forecasters to input the equation predictors through the use of slider bars and the probability of severe hail output is displayed. Figures 7, 8, and 9 show the GUI's for each of the equations.

A verification of the equations was performed using a relatively small independent dataset consisting of 53 cases. As with the developmental dataset, these data were derived from Local Storm Reports (LSR's) for the CAE CWA from April, 2011 through October, 2011 and from 2005 through 2006. These cases included 19 incidences of no hail, 11 were of small hail (diameter less than 1.00 in (2.5 cm)), and 23 were of severe hail (diameter 1.00 in (2.5 cm) or greater). Incidences of no hail were determined by arbitrarily selecting storms for which there were no storm reports. The Brier Score (BS; Brier 1950), which is a common method for measuring forecast accuracy of probability forecasts, was used to verify each equation. The values of the Brier Score range from zero to one with zero resulting from a perfect forecast. Therefore, a lower Brier Score is associated with a better forecast performance. The BS for LPSH100, LPSH0 and LPSH-20 were 0.1718, 0.1658 and 0.1582 respectively. In addition to the BS, the POD, FAR, and CSI were calculated for forecast severe hail probabilities ranging from 10 percent to 100 percent in 5 percent increments. Severe hail probability thresholds for each equation were obtained by determining which probability corresponded to the highest CSI. For LPSH100, the CSI was maximized for a severe hail probability output of 40 percent. For LPSH0 and LPSH-20, the CSI was maximized for a severe hail probability output of 45 percent. A summary of the results of the severe hail threshold calculations for each equation are presented in Table 3. A complete discussion of the BS, POD, FAR, and CSI can be found in Wilks (1995).

Hypothesis testing was conducted using the Pearson chi square goodness of fit test (X^2) to determine whether each model's predicted frequency (counts) of severe hail was consistent with the observed frequency (counts) in the independent sample. One of the advantages of using the chi square test is that a probability value (p-value) can be calculated to determine if the null hypothesis should be rejected or not. The null hypothesis was that the model's predicted frequency of severe hail and the observed frequency of severe hail were the same. Resulting p-values greater than the 0.05 significance level suggest that the null hypothesis should not be rejected and the models predictability of severe hail is dependable. A discussion of the Pearson chi square test can be found in Freeman (1987) and DeLisi (1998).

The results of the Pearson chi square goodness of fit test (X^2) are shown in Table 4. These results suggested not rejecting the null hypothesis in each case and also inferred that LPSH-20 was the most reliable equation for predicting severe hail followed by LPSH0.

Figures 10, 11 and 12 show the severe hail probability distributions for each equation from the independent verification sample and Table 5 summarizes the associated statistics. All three equations exhibited some degree of negative skewness with higher probabilities associated with the occurrence of severe hail. However, the distribution for LPSH100 showed more variability in the data and had the largest standard deviation. Furthermore, LPSH100 had the weakest Pearson chi square goodness of fit test result (with only two degrees of freedom). The weaker performance of LPSH100 may have been the result of not accounting directly for storm structure and updraft strength, whereas LPSH0 and LPSH-20 included the height of the 50 dBZ core as a primary predictor. The results of the statistical test and the meteorological reasoning support the conclusion that LPSH100 was the least statistically robust equation of the three in predicting severe hail.

4. Conclusions

Because the NWS criterion for severe hail was changed to a diameter greater than or equal to 1.00 inch (2.5 cm), there was a need to develop new objective methods to estimate the probability of severe hail during severe weather episodes. Local probability of severe hail equations were derived using logistic regression techniques. These equations included an equation to estimate a “VIL of the Day” and two additional equations that relate the height of the 50dBZ core above the 0^{o}C and -20^{o}C levels. The main purpose of the development of these equations was to improve the warning forecaster’s confidence of severe hail potential. Computer applications were developed to make the severe hail probability estimates readily available to the forecasters. The applications can be used prior to convection developing by providing estimates of threshold values that may aid in warning decision making. And if time allows, in conjunction with radar analysis as a supplement to visual interrogation of storm structure, the applications can be used to provide an objective method to estimate the probability of severe hail for individual storms.

Encouraging results were obtained from a verification of a relatively small independent sample. Statistical hypothesis testing suggested the derived equations were reliable estimates of the probability of severe hail and favorable Brier scores were attained suggesting accurate forecasts given the small sample. Based primarily on the results of the severe hail probability threshold analysis, it is recommended that operationally, the "VIL of the Day" be determined to be the value of the VIL in LPSH100 associated with a probability of severe hail of 40 percent. A severe hail probability output from LPSH0 and LPSH-20 of 45 percent is recommended for the threshold to consider a severe thunderstorm warning. However, verification on a larger sample will be needed to further quantify forecast skill and whether any modifications should be made to the severe hail probability threshold values. Additional cases are planned to be added to the developmental sample and re-derivations of the equations may result in improvement of the accuracy of the severe hail probability estimates.

Acknowledgments and Disclaimer

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Tables

Table 1. Pearson's Correlation Coefficients (r), probability-values (p-values), and degrees of freedom (df) for the potential predictor variables that had the strongest correlation with the dependent variable.

Potential Variable	***hgt50dbzfz***	***hgt50dbz20***	**VIL**	**VILD**
r	0.6663	0.6641	0.5932	0.5233
p-value	0	0	0	0
df	222	222	219	219

Table 2. Pearson's Correlation Coefficients (r), probability-values (p-values), and degrees of freedom (df) for the predictor variables in the LPSH models that exhibited strong collinearity.

Model	**LPSH100**	**LPSH0**	**LPSH-20**
Predictor Variables	VIL/VILD	*hgt50dbzfz*/VILD	*hgt50dbz20*/VILD
r	0.8497	0.7042	0.6967
p-value	0	0	0
df	219	222	222

Table 3. Probability of detection (POD), false alarm rate (FAR), and critical success index (CSI) for probability of severe hail output from each equation with the highest CSI's from the independent verification sample (53 cases). The severe hail probability thresholds chosen for each equation are shaded in yellow.

LPSH100		LPSH0		LPSH-20	
POD(60%)	0.52	POD(60%)	0.739	POD(60%)	0.65
FAR	0.2	FAR	0.29	FAR	0.29
CSI	0.462	CSI	0.566	CSI	0.517
POD(55%)	0.52	POD(55%)	0.783	POD(55%)	0.739
FAR	0.25	FAR	0.308	FAR	0.261
CSI	0.444	CSI	0.581	CSI	0.586
POD(50%)	0.609	POD(50%)	0.826	POD(50%)	0.783
FAR	0.26	FAR	0.296	FAR	0.25
CSI	0.5	CSI	0.613	CSI	0.621
POD(45%)	0.68	POD(45%)	0.913	POD(45%)	0.783
FAR	0.286	FAR	0.276	FAR	0.25
CSI	0.536	CSI	0.677	CSI	0.621
POD(40%)	0.739	POD(40%)	0.913	POD(40%)	0.826
FAR	0.261	FAR	0.323	FAR	0.296
CSI	0.586	CSI	0.636	CSI	0.613

Table 4. Summary of the Pearson chi square goodness of fit test (X^2) results to determine whether each model's predicted frequency (counts) of severe hail was consistent with the observed frequency (counts) in the independent sample (53 cases).

Model	LPSH100	LPSH0	LPSH-20
X^2	2.0797	1.6949	1.1633
p-value	0.3535	0.6380	0.7618
df	2	3	3

Table 5. Summary statistics for the severe hail equations from the independent sample (1 inch or greater cases) used for the verification. The Brier scores account for all the cases in the independent sample.

LPSH100		**LPSH0**		**LPSH-20**	
Mean	54.5217	**Mean**	71.8696	**Mean**	68.0435
Median	63	**Median**	73	**Median**	73
STD	27.1963	**STD**	24.0422	**STD**	26.8983
Brier Score	0.1718	**Brier Score**	0.1658	**Brier Score**	0.1582

Figures

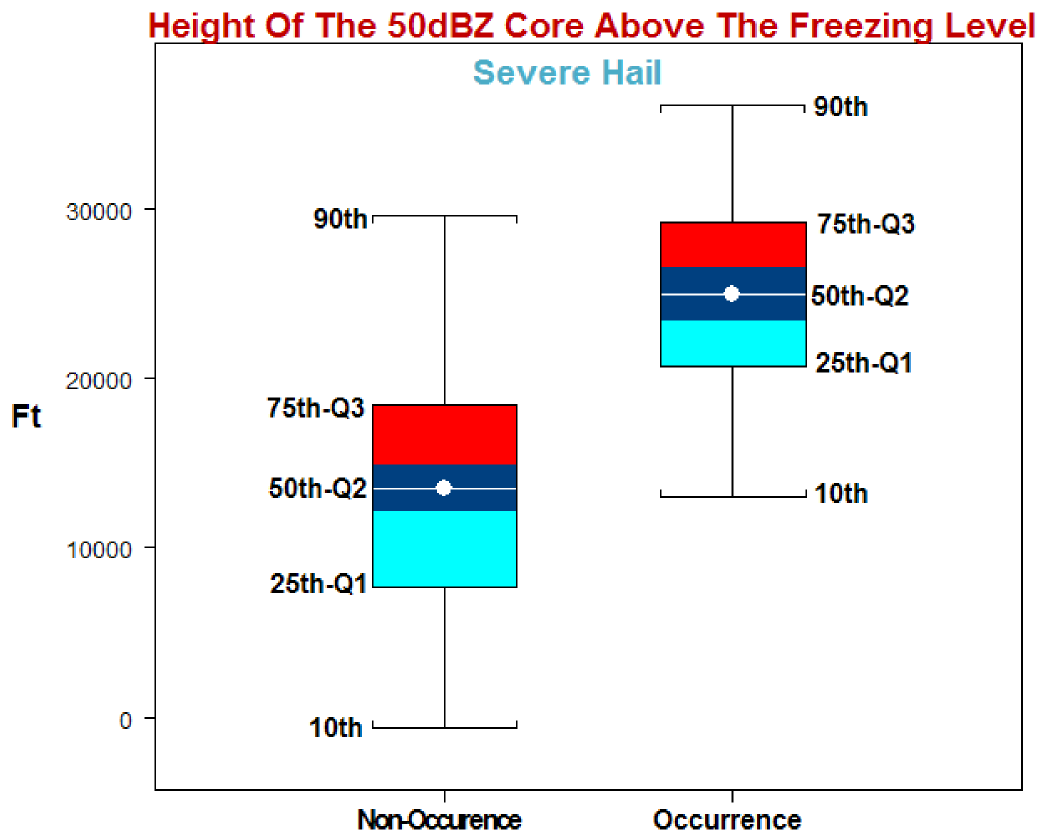

Figure 1. Box and whisker plots comparing the distributions between the height of the 50dBZ core above the 0^{o}C isotherm level variable (*hgt50dbzfz*) and the occurrence/non-occurrence of severe hail.

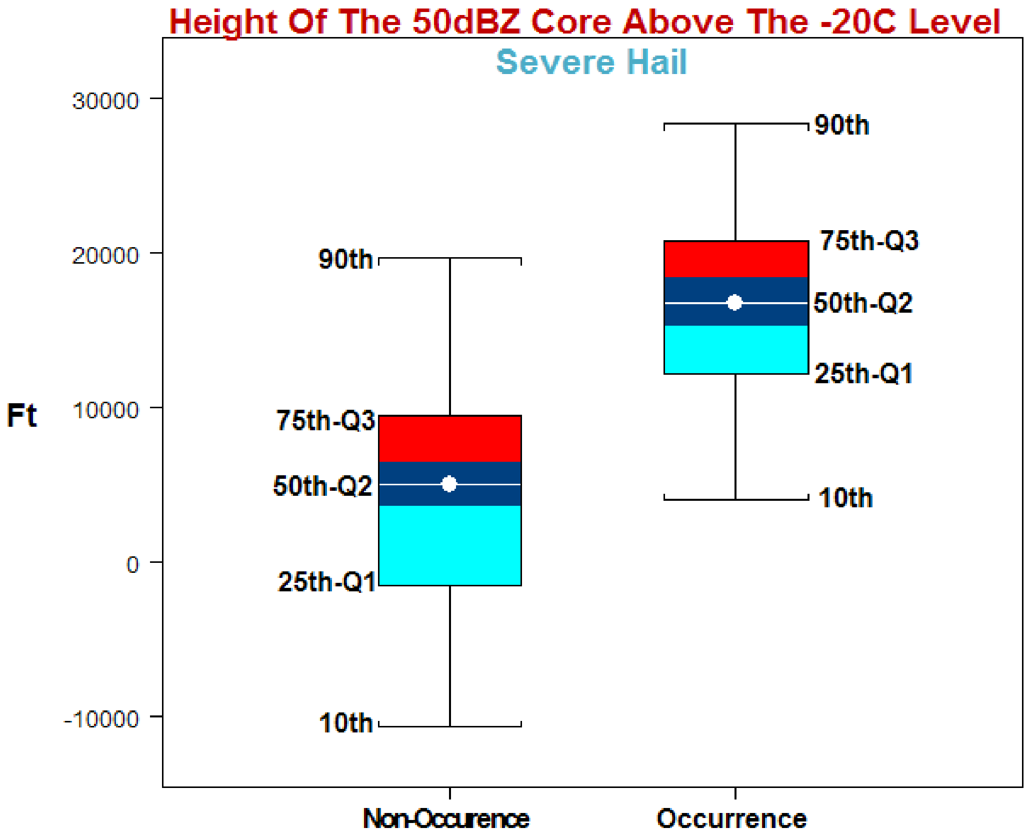

Figure 2. Box and whisker plots comparing the distributions between the height of the 50dBZ core above the -20^{o}C isotherm level variable (*hgt50dbz20*) and the occurrence/non-occurrence of severe hail.

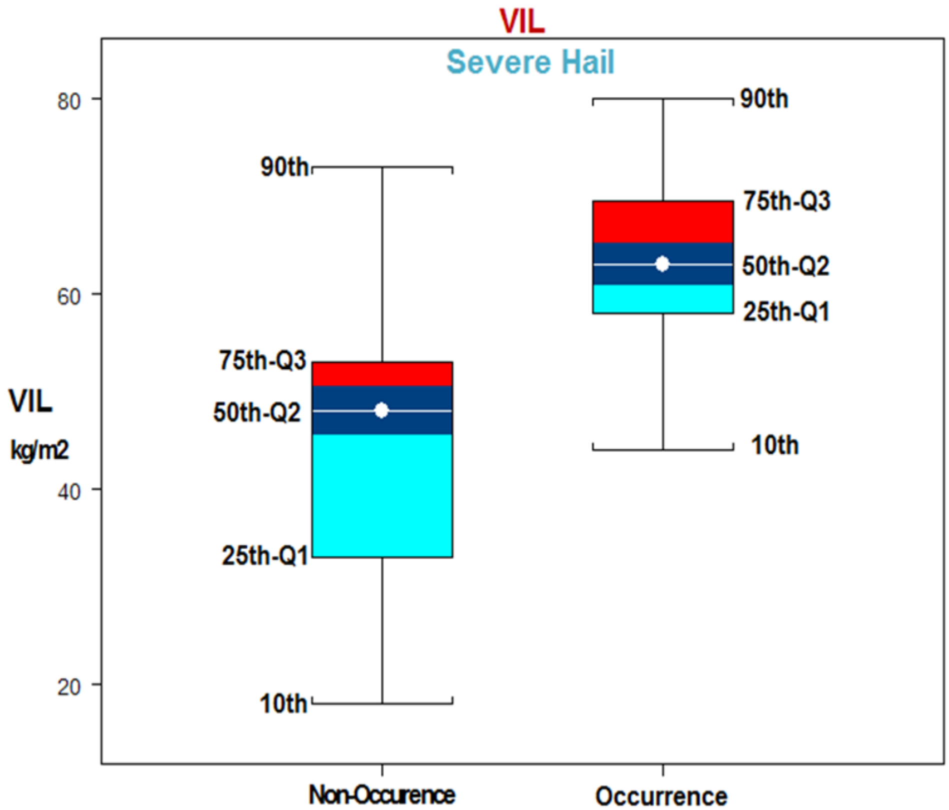

Figure 3. Box and whisker plots comparing the distributions between the vertically integrated liquid water variable (VIL) and the occurrence/non-occurrence of severe hail.

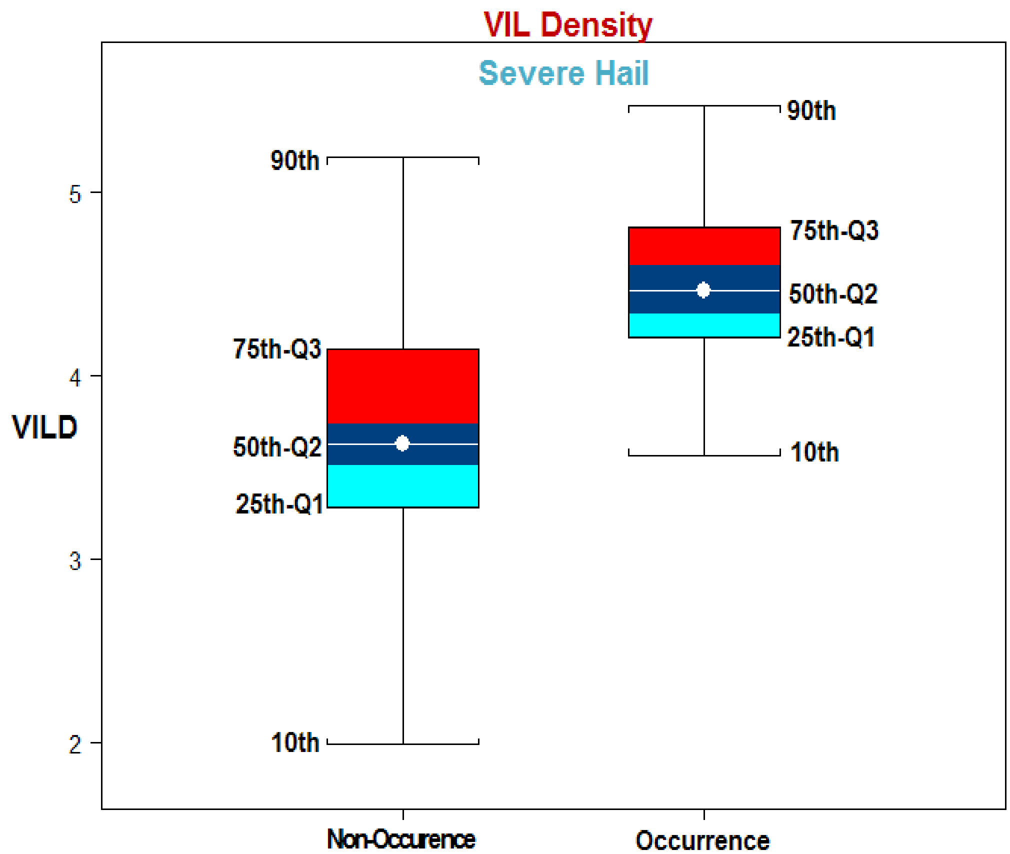

Figure 4. Box and whisker plots comparing the distributions between the vertically integrated liquid water density variable (VILD) and the occurrence/non-occurrence of severe hail.

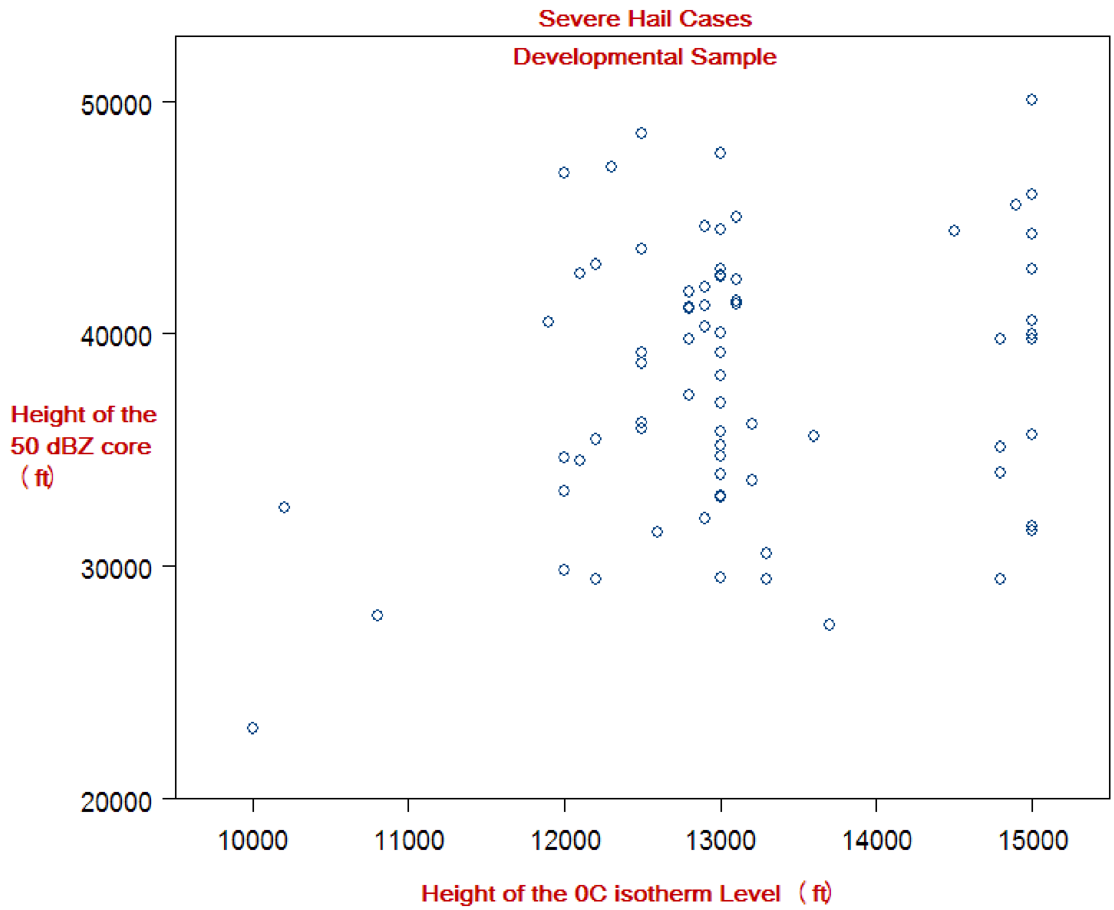

Figure 5. Scatter Plot showing the relationship between the height of the 50 dBZ core (ft.) and the height of the 0°C isotherm with regards to the severe hail cases in the developmental sample.

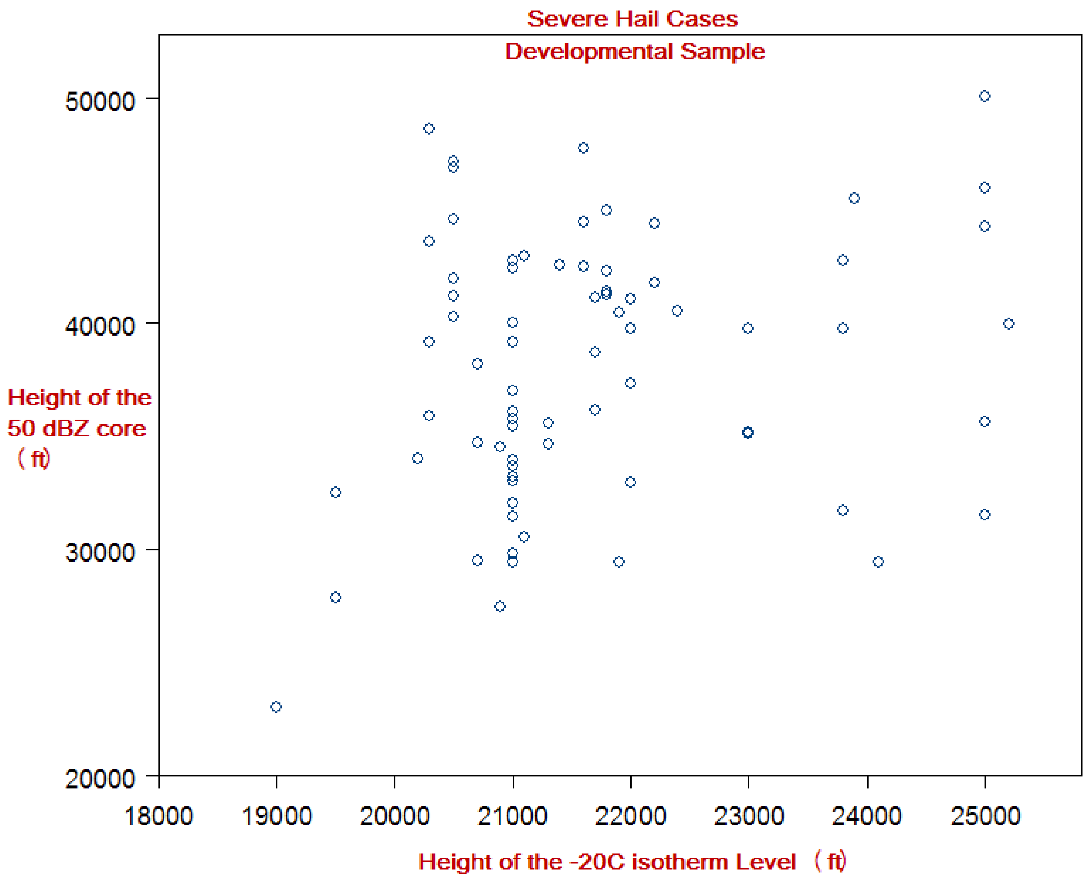

Figure 6. Scatter Plot showing the relationship between the height of the 50 dBZ core (ft.) and the height of the -20^{o}C isotherm with regards to the severe hail cases in the developmental sample.

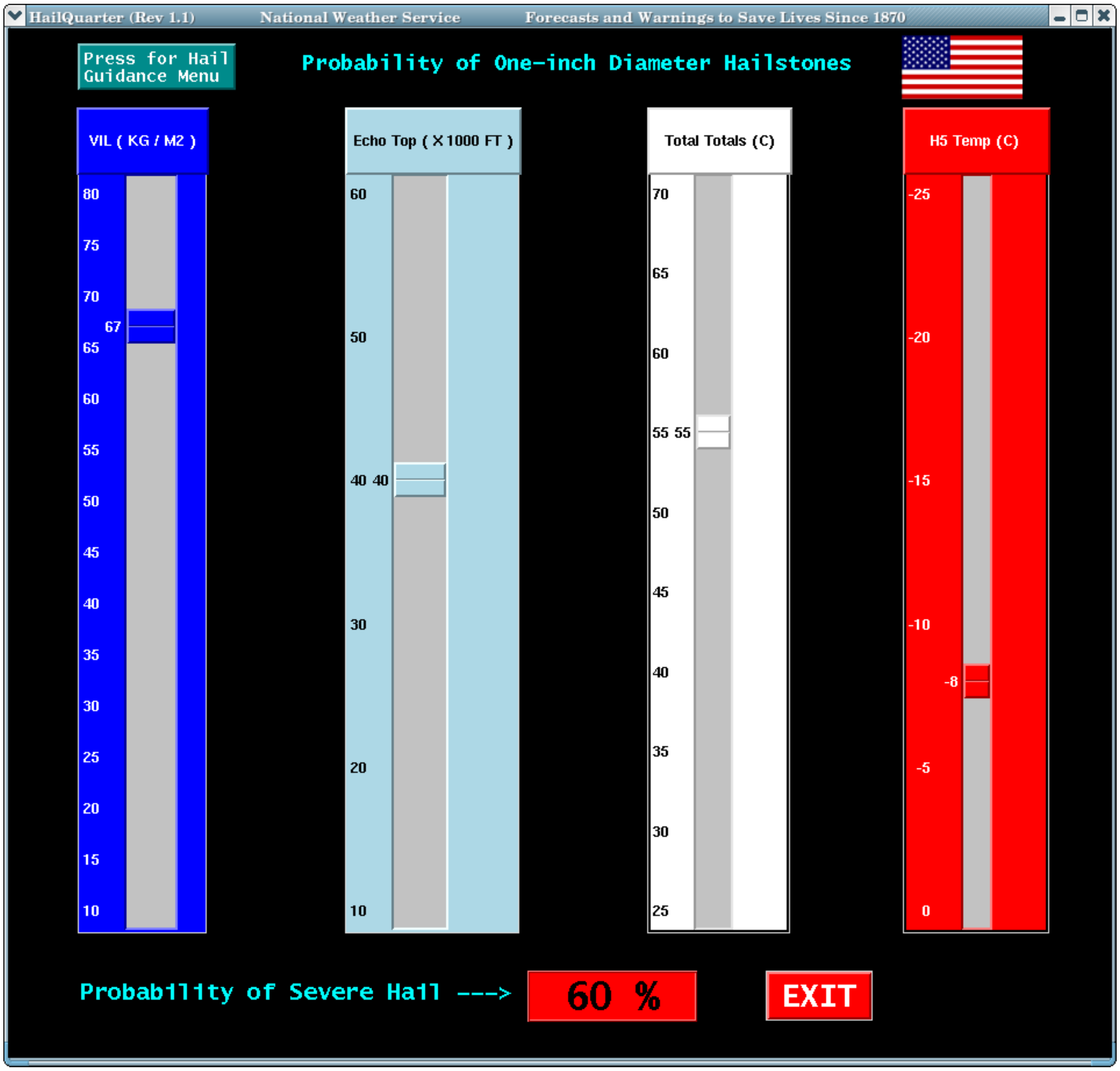

Figure 7. Computer application GUI for the LPSH100 "VIL of the Day."

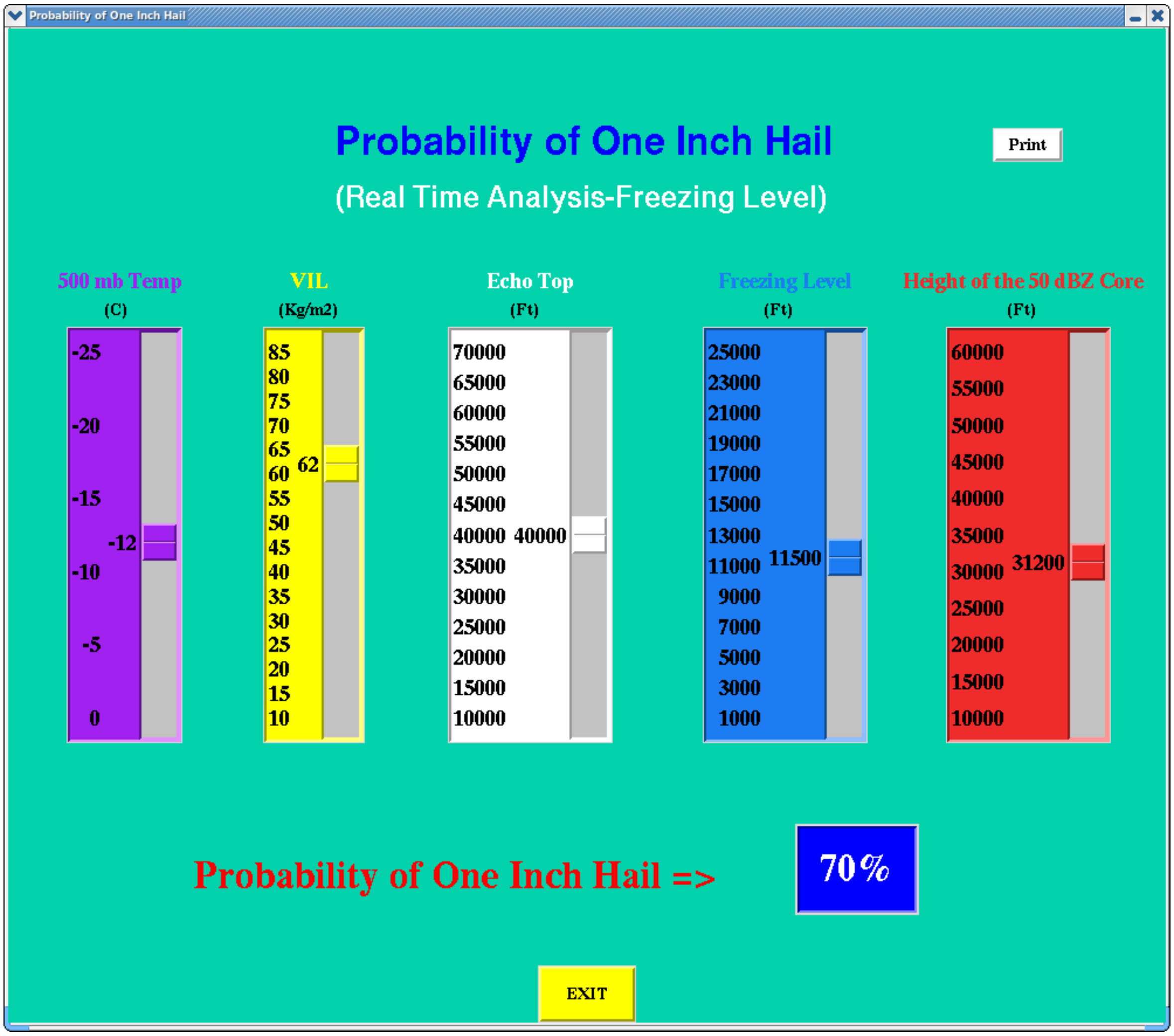

Figure 8. Computer application GUI for the LPSH0 (Height of the 50dBZ core above the 0^{o}C isotherm level).

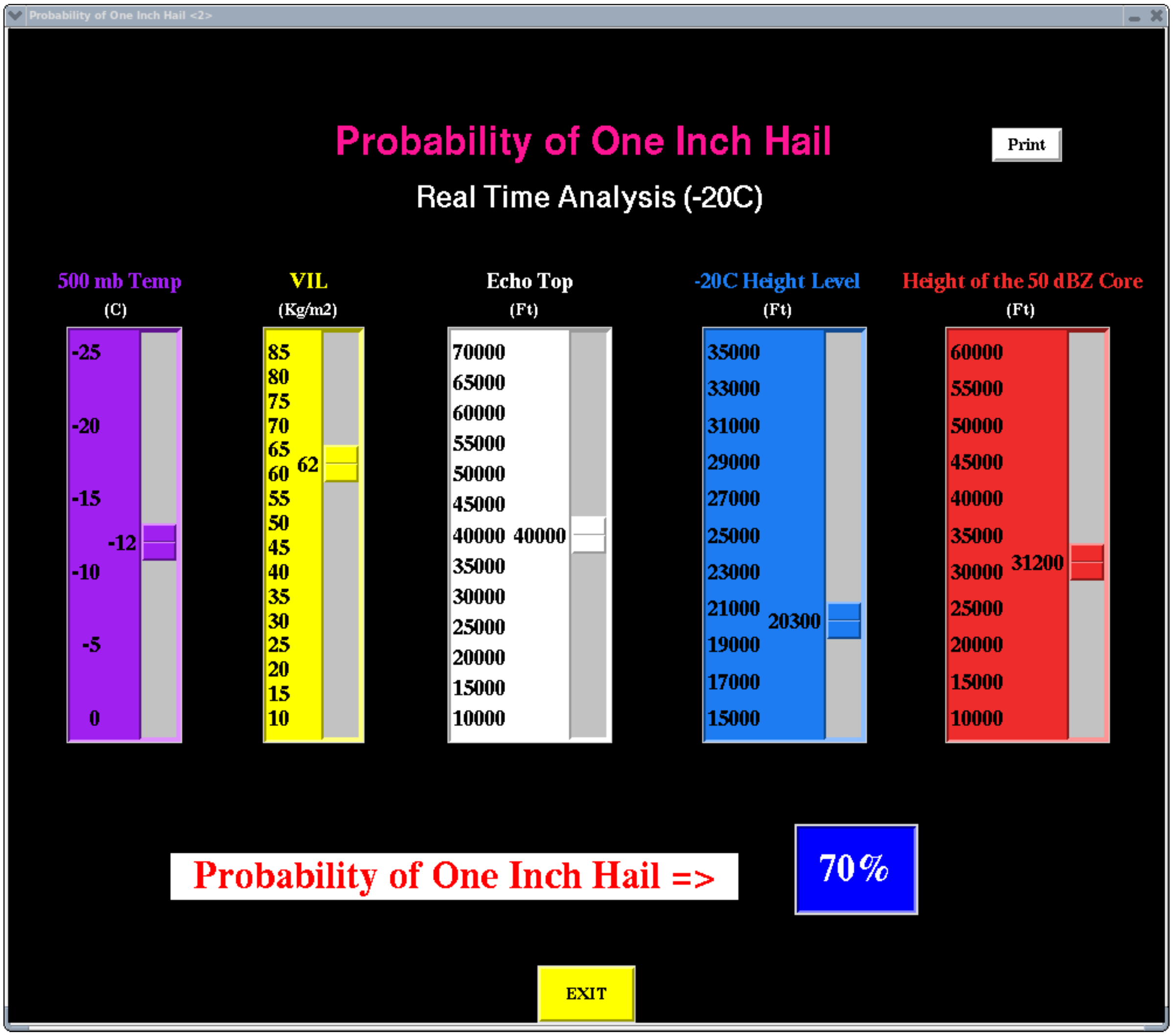

Figure 9. Computer application GUI for the LPSH-20 (Height of the 50dBZ core above the -20°C isotherm level).

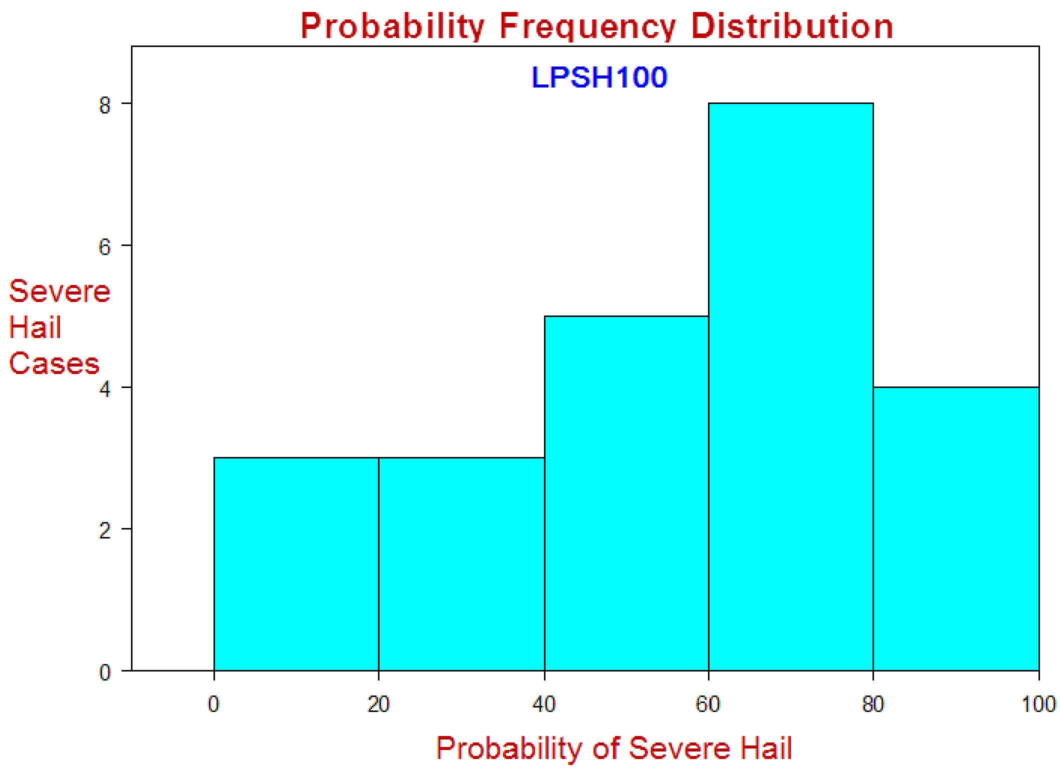

Figure 10. LPSH100 “VIL of the Day” severe hail probability distribution (23 cases) from the independent verification sample.

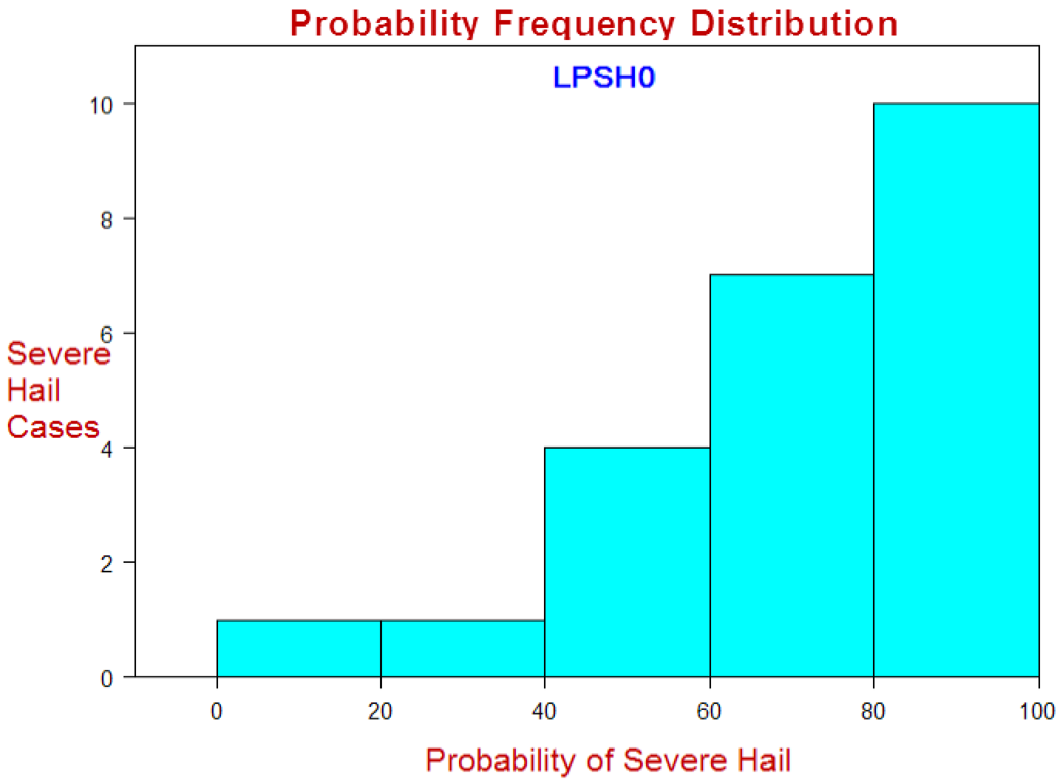

Figure 11. LPSH0 (Height of the 50dBZ core above the 0°C isotherm level) severe hail probability distribution (23 cases) from the independent verification sample.

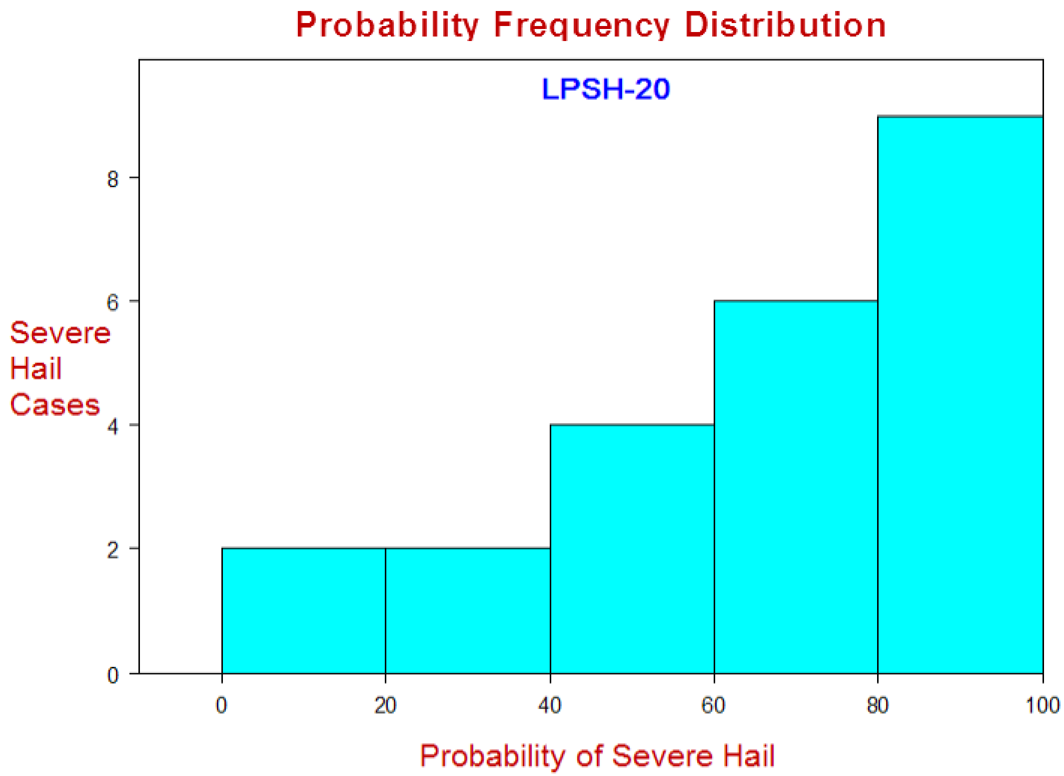

Figure 12. LPSH-20 (Height of the 50dBZ core above the -20^{o}C isotherm level) severe hail probability distribution (23 cases) from the independent verification sample.

Appendix A

Potential predictor variables included the following radar products from the Columbia, SC (KCAE) WSR-88D:

1) Vertically Integrated Liquid (VIL)
2) Vertically Integrated Liquid Density (VILD)
3) Enhanced Echo Tops (ET)
4) Height of the 50dBZ Reflectivity Core (above the 0^{o}C isotherm level; *hgt50dbzfz* and above the -20^{o}C isotherm level; *hgt50dbz20*)

Potential predictor variables included the following from the Rapid Update Cycle (RUC) model and SPC meso-analysis:

1) Wet-Bulb Zero Height (WBzero)
2) Bulk Richardson Number (BRN)
3) 6-km Shear (SHEAR6km)
4) K-Index (KI)
5) Showalter Index (SW)
6) Equilibrium Level (EQLVL)
7) -20^{o}C Isotherm Level (hgt20C)
8) Convective Available Potential Energy -10^{o}C to -30^{o}C (CAPE1030C)
9) Precipitable Water (PW)
10) Craven Brooks Significant severe parameter (SIGSVR)
11) Significant Hail Parameter (SIGHAIL)
12) Supercell Composite Parameter (SCP)
13) Low Level Lapse Rate (LAPSERATE)
14) 500-hPa Temperature (H5TEMP)
15) Freezing Level: 0^{o}C Isotherm Level (FZLEVEL)
16) 100-hPa Mean Layer Convective Available Potential Energy (MLCAPE)
17) Lifted Index (LI)
18) Total Totals Index (TT)